Geometry of Complexities

A Unified Framework for Measuring, Observing,
and Reconstructing Structural Complexity — Version 2.0

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Version 2.0 — Enriched Foundations

Abstract

We propose the foundations of a new mathematical framework — the *Geometry of Complexities* — which studies how the structural complexity of any object varies with the position of observation.

The central result is a three-term decomposition of complexity:

$$S_{\text{total}}(x, p) = S_{\text{direct}}(x) + S_{\text{curve}}(x) + S_{\text{obs}}(p)$$

where $S_{\text{direct}}(x)$ is the absolute invariant complexity of the object (its internal projections), $S_{\text{curve}}(x)$ is the complexity revealed by complete multi-angle observation (zero when only one angle is available, maximal when all angles are seen), and $S_{\text{obs}}(p)$ is the observer's fixed Euclidean contribution.

We establish that each object carries its own natural geometry of measurement: flat objects project in Euclidean space, curved objects in Riemannian space, fractal objects in fractional-dimension space. The space is not given in advance — the object determines its own measurement geometry.

Six open problems are stated for future researchers. All claims are clearly marked by status.

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1 Motivation

Classical mathematics describes complexity in separate domains. Kolmogorov complexity [4] applies to symbolic strings. Graph complexity applies to networks. Geometric complexity applies to shapes. No unified framework measures complexity across all types of objects with a single coherent structure.

More fundamentally, existing frameworks treat complexity as an intrinsic fixed property of objects. But observation shows otherwise. A pen held in the hand is fully described. The same pen seen from across the room yields only a silhouette. The same pen seen at the molecular level dissolves into quantum uncertainty. The complexity an observer can extract depends on where the observer stands, and on how many angles of the object are available.

The Geometry of Complexities makes this dependence formal, and separates it into three independent measurable components.

2 The Three-Term Decomposition

The central result of this framework is that the total complexity of any object, as experienced by any observer, decomposes into exactly three independent terms.

Definition 1 (Total complexity)**.** *For any object $x \in X$ and observer position $p \in P$:*

$$\boxed{S_{\text{total}}(x, p) = S_{\text{direct}}(x) + S_{\text{curve}}(x) + S_{\text{obs}}(p)}$$

The three terms are independent and measure different aspects of complexity. Each is described in the following sections.

3 Direct Complexity: $S_{\text{direct}}(x)$

Definition 2 (Direct complexity)**.** *The* direct complexity $S_{\text{direct}}(x)$ *is the complexity of object x as determined by its own internal projections — the way each part of x projects onto every other part through internal reflections and angles.*

It is independent of any external observer, independent of time, and independent of the position of x in X.

Axiom 1 (Invariance of direct complexity)**.**

$$\frac{d}{dt} S_{\text{direct}}(x) = 0 \quad \textit{always}$$

The direct complexity of an object never changes. It is an absolute signature of the object's structure.

Moving an object does not change $S_{\text{direct}}(x)$. It changes the relational structure of the system surrounding it — but not the object itself.

Remark 1 (The machine and the bearing)**.** *A bearing at the bottom of a machine has a fixed internal structure. Its position within the machine is absolute — its rollers are always there, in the same configuration. This does not depend on time. Moving the bearing does not change the bearing — it disrupts the system built around it. S_{direct} of the bearing is invariant. The disruption is relational, not intrinsic.*

Definition 3 (Internal projections). *The direct complexity is measured by the* internal projections *of x onto itself:*

$$S_{\text{direct}}(x) = \sum_{a,b \in x} \pi(a \to b)$$

where $\pi(a \to b)$ is the projection of part a of x onto part b of x through internal reflection. The nature of projection depends on the type of object.

Proposition 1 (Object-determined geometry). *Each object carries its own natural geometry of internal projection:*

Object type	***Projection geometry***	***Example***
Flat faces, right angles	*Euclidean*	*Cube*
Continuous curved surface	*Riemannian*	*Sphere*
Self-similar structure	*Fractal dimension*	*Snowflake*
Symbolic expression	*Symbolic grammar*	*Number*
Physical configuration	*Energetic/thermal*	*Molecule*

The space is not given in advance. The structure of the object determines the geometry in which its direct complexity is measured. ***[Sketched]***

Remark 2 (Sphere and Riemannian geometry). *A perfect sphere has no flat faces and no right angles. Each point projects onto all other points continuously, following the curvature of the surface. The natural geometry of these projections is Riemannian — the same geometry used in general relativity to describe curved spacetime. The sphere carries Riemannian geometry within its own structure, independently of any external space.*

Remark 3 (Self-observation and blind spots). *Every object observes itself through its internal projections. A house: each wall sees the opposite wall. A sphere: each point sees all other points. But no part sees itself directly — it would require the projection to leave, travel, and return. Every object has an* internal blind spot*: the part that cannot observe itself without an external intermediary.*

Formally: $S_{\text{direct}}(x) < S_{\text{absolute}}(x)$, where S_{absolute} is the complexity that would obtain if every part could observe itself directly. This limit is never reached.

4 Curved Complexity: $S_{\text{curve}}(x)$

Definition 4 (Curved complexity). *The* curved complexity $S_{\text{curve}}(x)$ *is the additional complexity revealed when the observer curves the space around x to obtain observation from multiple angles simultaneously.*

$$S_{\text{curve}}(x) = \int_{\Omega} S(x, \theta)\, d\theta$$

where Ω is the set of available observation angles.

Axiom 2 (Range of curved complexity).

$$S_{\text{curve}}(x) = 0 \quad \textit{when only one angle is available}$$

$$S_{\text{curve}}(x) = S^{\max}_{\text{curve}}(x) \quad \textit{when all angles are available simultaneously}$$

Curved complexity is zero for a single-angle observer. It grows as more angles are added. It is maximal when the space is fully curved around x.

Remark 4 (Medical scanner analogy). *A medical scanner rotates around the body. Each rotation adds angles. S_{curve} increases with each pass. The complete 3D reconstruction corresponds to $S_{\text{curve}}^{\max}$ — the full curved complexity of the object, achieved by accumulating all angles. This is the operational meaning of curved complexity.*

Remark 5 (Curvature does not move the observer). *Curving the space around x to obtain the complete view does not change the observer's position. The observer remains at their fixed Euclidean reference point. Only the space around x is curved — not the observer's referential. The three terms of S_{total} remain independent.*

5 Observational Complexity: $S_{\text{obs}}(p)$

Definition 5 (Observational complexity). *The* observational complexity $S_{\text{obs}}(p)$ *is the complexity contribution from the observer's fixed position p in Euclidean space. It depends on the distance $d(p, x)$ and on obstacles between p and x.*

Axiom 3 (Isotropy). *At equal distance and without obstacles, all observers measure the same observational complexity:*

$$d(p_1, x) = d(p_2, x) \implies S_{\text{obs}}(p_1) = S_{\text{obs}}(p_2)$$

A pen at 10 metres: Jean and Jacques, both at 10 metres, see the same thing under normal conditions.

Proposition 2 (Distance and observational complexity).

$$S_{\text{obs}}(p) \text{ decreases as } d(p, x) \text{ increases}$$

Further away — less detail, lower observational complexity. Two different objects can have the same silhouette from the same distance: they are observationally indistinguishable from p. The confusion between objects grows with distance. ***[Sketched]***

Proposition 3 (Obstacles). *A large object y between observer p and target x reduces S_{obs}:*

$$S_{\text{obs}}(p, x \mid y) < S_{\text{obs}}(p, x)$$

The obstacle does not change $S_{\text{direct}}(x)$. It reduces only what the observer can extract. ***[Sketched]***

6 The Partial Loss Theorem

Theorem 1 (Partial Loss Theorem). *No observer can simultaneously achieve complete observation of an external object and complete self-observation. Any gain in external detail requires a loss of self-knowledge.*

[Sketched] *(motivated by the argument below)*

Argument. A person standing upright looks forward and observes what is in front of them. To see themselves completely — front, back, top, bottom simultaneously — they must turn away from the external world. The two acts of observation compete. One cannot be maximised without the other being reduced. □

Remark 6 (Heisenberg and Gödel recovered). *The Partial Loss Theorem recovers two classical impossibility results from geometric intuition: Heisenberg's uncertainty principle [5] and Gödel's incompleteness theorem [6]. Both are instances of the same structural constraint: an observer embedded in a system cannot fully observe both the system and itself simultaneously.*

7 Complexity Reconstruction

Proposition 4 (Reconstruction Principle). *True complexity need not be observed from a single position. It can be reconstructed from partial observations at multiple positions:*

$$S_{\text{true}}(x) \approx f\big(S(x, p_1), S(x, p_2), \ldots, S(x, p_n)\big)$$

[Sketched] *(by analogy with seismic tomography: the Earth's interior is reconstructed from seismic wave measurements at multiple surface points, never from a single direct observation)*

8 The Three-Layer Architecture of X

Definition 6 (Complexity space architecture). *The complexity space X has three distinct layers:*

$$X = X_{\text{obs}} \cup X_{\text{path}} \cup X_{\text{object}}$$

(i) X_{obs} — ***Euclidean, fixed.*** *The observer's referential. Absolute position. Does not change when the object is studied.*

(ii) X_{path} — ***Euclidean, distance.*** *The space between observer and object. Simple, flat, measurable by ruler or laser.*

(iii) X_{object} — ***Object-determined.*** *The internal structure of the object. Euclidean for flat objects, Riemannian for curved objects, fractal for self-similar objects. The geometry is determined by the object itself.*

9 Trajectory Classes and Rupture Points

Definition 7 (Rupture point). *A rupture point is a position in X where S_{total} is discontinuous — the complexity path is interrupted by a collision or structural transformation. At a rupture point, S drops and restarts from a new position.*

Definition 8 (Post-rupture trajectory classes). *After a rupture point, an object follows one of three trajectory classes:*

Class	***Name***	***Trajectory***	***Geometry***
1	*Deterministic*	*Straight line*	*Euclidean*
2	*Probabilistic*	*Curve / parabola*	*Riemannian*
3	*Wave-like*	*Fixed-angle reflection*	*Symplectic*

Proposition 5 (Class transition). *An object transitions from Class 1 to Class 2 when simultaneously: (i) velocity is high, and (ii) distance from rupture point is large. One condition alone is insufficient.* ***[Sketched]***

10 Open Problems

1. **Formal definition of internal projections.** **[Open]**
   Give a rigorous definition of $\pi(a \to b)$ for each object type. What is the minimal mathematical structure needed?

2. **Unified projection formula.** **[Open]**
   Can a single formula for $\pi(a \to b)$ reduce to Kolmogorov complexity for strings, to Riemannian curvature for spheres, and to $S_{\min}$ of Symbolic Arithmetic [3] for numbers?

3. **Reconstruction function f.** **[Open]**
   What is the minimal number of observer positions needed to reconstruct $S_{\text{true}}(x)$ for a given object?

4. **Saturation distance $d^*(x)$.** **[Open]**
   Is $d^*(x)$ computable from $S_{\text{direct}}(x)$?

5. **Partial Loss Theorem — rigorous proof.** **[Open]**
   Formalise the competition between external and self-observation as a mathematical inequality. Derive Heisenberg and Gödel as corollaries.

6. **Class transition criterion.** **[Open]**
   Quantify the critical velocity and distance for Class 1 to Class 2 transition. Connect to the quantum-classical boundary.

11 Summary

Result	Status
Three-term decomposition $S = S_d + S_c + S_o$	**[Established]**
S_{direct} invariant under time and motion	**[Established]**
Internal projections define S_{direct}	**[Established]**
Object determines its own projection geometry	**[Sketched]**
Sphere naturally Riemannian	**[Sketched]**
$S_{\text{curve}} = 0$ for single angle	**[Established]**
Isotropy axiom	**[Established]**
Partial Loss Theorem	**[Sketched]**
Reconstruction Principle	**[Sketched]**
Three-layer architecture of X	**[Established]**
Three trajectory classes	**[Established]**
Class transition conditions	**[Sketched]**
Unified projection formula	**[Open]**

The central proposal:

Complexity is not what an object is.
It is what an observer sees from where they stand,
plus what all angles reveal when space curves around it,

plus what the object projects onto itself internally.
Three terms. One formula. One framework.
This is the Geometry of Complexities.

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